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Reconnection of colliding cosmic strings

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Abstract: For vortex strings in the abelian Higgs model and D-strings in superstring theory, both of which can be regarded as cosmic strings, we give analytical study of reconnection (recombination, inter-commutation) when they collide, by using effective field theories on the strings. First, for the vortex strings, via a string sigma model, we verify analytically that the reconnection is classically inevitable for small collision velocity and small relative angle. Evolution of the shape of the reconnected strings provides an upper bound on the collision velocity in order for the reconnection to occur. These analytical results are in agreement with previous numerical results. On the other hand, reconnection of the D-strings is not classical but probabilistic. We show that a quantum calculation of the reconnection probability using a D-string action reproduces the nonperturbative nature of the worldsheet results by Jackson, Jones and Polchinski. The difference on the reconnection — classically inevitable for the vortex strings while quantum mechanical for the D-strings — is suggested to originate from the difference between the effective field theories on the strings.

Keywords: Solitons Monopoles and Instantons, D-branes, Tachyon Condensation, Cosmology of Theories beyond the SM.

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1. Introduction

Recent revival of the study of cosmic strings [1] originates partially in the proposal that cosmic strings can be fundamental superstrings, D-strings, or their bound states called (p, q)-strings in various compactification scenarios [2, 3].[1] Many other options like wrapped branes on supersymmetric cycles or tensionless strings can be good candidates as well. Though first mostly denied by E. Witten [5] 20 years ago, this possibility was re-born thanks to the decade's developments in string/M-theory which have provided fertile bases for constructing semi-realistic universes in terms of D-branes. Pursuing the possibility is quite important by the obvious reason that we may observe some signals of those very stringy objects directly in the sky.

For cosmic strings, various classical solutions such as vortex strings in abelian Higgs models and other scalar/gauge field theories allowing topological vortices have been adopted to model them and study their properties.[2] One of the points which distinguishes the fundamental strings or the D-strings from the field theory vortex strings is in their *reconnection*

[1] See also reviews [4].

[2] Note that Type I/II cosmic strings are not related to Type I/II superstrings. The former types are distinguished by a relation between the two coupling constants in the abelian Higgs model. In addition, "gauge strings" refer to vortex strings in the abelian Higgs model, while "global strings" arise in scalar field theories with topologically nontrivial vacua.

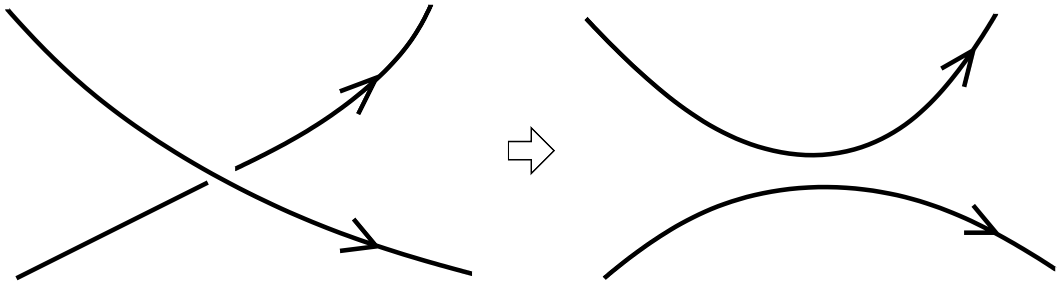

Figure 1: Reconnection of strings.

probability [3]. When strings collide with a relative angle, they may be cut once at the collision and connected at the different ends. This is called reconnection (or recombination, inter-commutation), see figure 1. This reconnection probability is one of the indispensable ingredients for simulating galaxy formation in the early universe, and is important also for the direct detection of gravitational waves arising from cusps created when the cosmic strings are reconnected. For the vortex strings, numerical simulations [6 − 8] have been extensively performed and exhibit the universal feature: *these vortex strings always reconnect classically for small collision velocities, while above a velocity upper bound they do not reconnect.* On the other hand, *for fundamental strings and D-strings, the reconnection is probabilistic for any collision velocity* [9]. Based on the difference between the two, some numerical simulations depending on this probability were reported recently [10].

In this paper, we clarify the origin of this difference theoretically and analytically. In a word, it comes from the difference of the effective field theories on the strings. For the effective action of the D-strings (which is called a D-string (or D-brane) action) there are classical solutions representing D-strings passing through each other without reconnection. Since such classical solutions exist, the reconnection occurs only in a probabilistic manner.[3] On the other hand, for vortex strings the effective field theory on those does not allow such classical solutions and thus the reconnection takes place classically, and inevitably.

The effective field theory on the vortex is a sigma model whose target space has been derived recently by using brane realization techniques [12]. Thus it comes from a D-brane action, but the presence of additional matter fields due to the specific brane configuration and also the field theory limit make the resulting effective field theory of the vortex strings look very different from the usual D-brane action. More precisely, the theory on the vortex string contains fundamental matter fields and the relevant dynamics is in the Higgs phase of the effective theory. We give an analytical proof that the vortex strings always reconnect for small collision velocity and collision angle, by using the effective sigma model. This technique also allows to explain the existence of the velocity upper bound, and we derive it analytically by considering a geometrical constraint on the shape of the reconnected vortex strings. It turns out that the upper bound coincides with the result of [13] which was obtained by looking at deformations of classical solutions of the abelian Higgs model.

[3]For fundamental strings, the reconnection is probabilistic from the first place as known in an interaction in a string field theory [11]. So we don't deal with the fundamental strings in this paper.

For colliding D-strings, classically nothing occurs because the usual D-string action allows a simple solution describing the D-strings passing through each other without reconnection. But quantum mechanically there appears a tachyonic instability intrinsic to intersecting D-branes [14, 15]. The tachyon condensation leads to the reconnection [16], and we study time evolution of a tachyon wave function to evaluate the reconnection probability. The result shares the same non-perturbative property as found in the string worldsheet calculation in [9].

The organization of this paper is as follows. In section 2, we show two effective actions and give the classical difference between the colliding solutions, to explain the difference between the reconnection property of the D-strings and the reconnection property of the vortex strings. In section 3, we give the analytical proof of the reconnection of the vortex strings and the velocity upper bound. In section 4, the D-string reconnection probability is calculated. Section 5 is devoted to a summary and discussions.

2. Vortex strings and D-strings

The abelian Higgs model of our interest can be realized as a field theory limit of a theory on a D4-brane embedded in a certain brane configuration [17], and therefore the vortex strings can also be described as a D-brane in that brane configuration [12]. We shall review the D-brane action and the brane configuration, in order to clarify the difference between the vortex strings and the D-strings themselves.

First, we consider D-strings in the absence of other kinds of branes. The D-strings are moving in 10 dimensional spacetime, and in some compactification scenario these D-strings can be thought of as cosmic strings. The bosonic part of the D-string low energy action in flat target spacetime is

$$S = \frac{2\pi l_{\rm s}^2}{g_{\rm s}} \int dtdx \ \mathrm{Tr}\left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} D_\mu \Phi_i D^\mu \Phi^i + \frac{1}{4}[\Phi_i, \Phi_j]^2 \right] \tag{2.1}$$

where the eigenvalues of $2\pi l_{\rm s}^2 \Phi$ ($l_{\rm s}$ is the string length) measures the target space location transverse to the worldsheet of the D-strings, and thus i runs from 2 to 9, the transverse dimensions. We consider a pair of D-strings tilted and colliding with each other, and in fact there is a classical solution representing them passing through each other without reconnection:

$$2\pi l_{\rm s}^2 \Phi_2 = \begin{pmatrix} \tan\left(\frac{\theta}{2}\right) x & 0 \\ 0 & -\tan\left(\frac{\theta}{2}\right) x \end{pmatrix}, \qquad 2\pi l_{\rm s}^2 \Phi_3 = \begin{pmatrix} \overline{v} t & 0 \\ 0 & -\overline{v} t \end{pmatrix}. \tag{2.2}$$

Here θ is the relative angle between the D-strings, and $2\overline{v}$ is the relative velocity.[4] At the collision incidence, nothing like the reconnection occurs. The solution represents two straight D-strings, but any collision of D-strings can be described by this solution at least locally around the collision point. This immediately tells us that colliding D-strings do not

[4] The action (2.1) is valid for a slow motion of the D-strings. To treat fast (relativistic) collisions, one has to use the non-abelian Born-Infeld action.

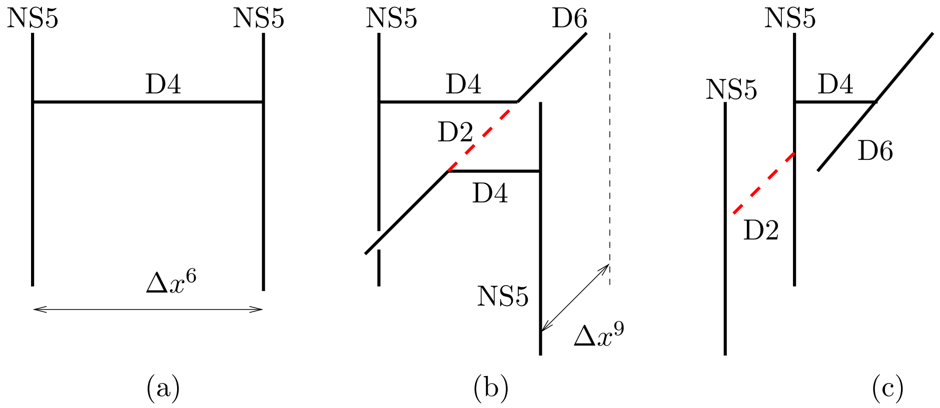

Figure 2: The brane configuration relevant for the abelian Higgs model. (a) On the D4-brane suspended between parallel NS5-branes, 4 dimensional $\mathcal{N} = 2\,\mathrm{U}(1)$ theory is realized. (b) The FI term is turned on, and the effective theory is the abelian Higgs model. The dashed line ending on the D4-branes shows a D2-brane (on a "flavor" D6-brane) which is identified with a vortex string. (c) The D2-brane (dashed line) is in a Coulomb phase of its effective field theory. Because it can freely move away from the D4-brane, there is no solitonic interpretation in 3+1 dimensions.

cause the reconnection classically.[5] This argument is valid as long as the D-string action is effective: for decoupling of closed strings ($g_s \to 0$) and at the low energy ($\theta \ll 1$ and $\bar{v} \ll 1$ for the configuration of (2.2)).

On the other hand, we claim that any collision of vortex strings in the abelian Higgs model causes the reconnection if the collision velocity and the collision angle are small enough, that is, for the same parameter region ($\theta \ll 1, \bar{v} \ll 1$). We first derive the effective theory on the pair of the vortex strings by following [12], then show that any classical solution of the form (2.2) does not exist. The derivation in [12] is through brane configurations in Type IIA string theory, and resultantly the effective action comes from a D-brane action and thus resembles (2.1). However, as we will see, there appears a crucial difference.

The $\mathcal{N} = 2$ abelian Higgs model in 4 dimensions can be realized as a theory on a D4-brane suspended between two parallel NS5-branes in Type IIA superstring theory: see figure 2(a). We have in addition several D6-branes perpendicular to the other branes, to realize hypermultiplets. Turning on the Fayet-Illiopoulos parameter v_{AH} corresponds to a translation of one of the NS5-branes along x^9. The vortex strings are realized as D2-branes placed on the D6-branes and suspended between the D4-branes, see figure 2(b). These D2-branes correspond to the vortex strings, as they possess the correct charges and amount of supersymmetries. The bosonic part of the effective field theory on two D2-branes is [12]

$$S_{\mathrm{vortex}} = \int dt dx\, \mathrm{Tr}\left[-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - \mathcal{D}_\mu Z^\dagger \mathcal{D}^\mu Z - \mathcal{D}_\mu \psi^\dagger \mathcal{D}^\mu \psi - \frac{g^2}{2}\left(\psi\psi^\dagger - [Z, Z^\dagger] - r\mathbb{1}_{2\times 2}\right)^2\right] \tag{2.3}$$

[5] However, a quantum treatment shows probabilistic reconnection via condensation of tachyonic fundamental strings connecting the two D-strings, as we will see in section 4.

which is a $1+1$ dimensional U(2) gauge theory with a complex adjoint field Z and a vector field ψ. Note that the theory becomes 2 dimensional since we took a limit where the dynamics along x^9 can be ignored. We appropriately rescaled the adjoint scalar fields Φ^i and introduced a complex field representation $Z \propto \Phi^2 + i\Phi^3$. The lagrangian includes terms of the D-string action (2.1), but two features peculiar to (2.3) is that there are a FI parameter r and a new field ψ. This r and the gauge coupling appearing in the effective theory are related to the string theory parameters and the original abelian Higgs model parameters by

$$\frac{1}{g^2} = \frac{l_{\rm s}\Delta x^9}{g_{\rm s}} = (2\pi)^3 l_{\rm s}^4 v_{\rm AH}^2\,, \qquad r = \frac{\Delta x^6}{2\pi g_{\rm s} l_{\rm s}} = \frac{2\pi}{g_{\rm AH}^2}\,. \tag{2.4}$$

Here $v_{\rm AH}$ and $g_{\rm AH}$ are the FI parameter and the gauge coupling, respectively, in the original abelian Higgs model. As shown in [12], we have to take the strong coupling limit $g \to \infty$ so that the original abelian Higgs model is decoupled from other stringy modes. Therefore, the dynamics of the vortex strings is dictated solely by the moduli space of the theory (2.3). This moduli space is defined by the D-term condition

$$\psi\psi^\dagger - [Z, Z^\dagger] - r\mathbb{1}_{2\times 2} = 0\,. \tag{2.5}$$

Thus, the effective theory of the vortex strings is a sigma model whose target space is defined with this D-term equation. It describes any slow motion of the vortex strings.[6]

The important fact is that the D-term equation (2.5) does not allow the configuration (2.2) due to the existence of the FI parameter r. The configuration (2.2) gives vanishing commutator $[\Phi_2, \Phi_3] = 0$ which cannot satisfy (2.5) because the 2×2 matrix $\psi\psi^\dagger$ is of rank 1. Thus, there is no naive classical solution of vortex strings passing through each other.

If we take $r = 0$, the lagrangian (2.3) mostly reduces to the D-strings action (2.1). In the brane configuration, this may be achieved by putting $\Delta x^6 = 0$, then the D4- and D6-branes become irrelevant for the motion of the D2-branes, and thus there are no fundamental fields and no FI term in the D2-brane action. In this limit $\Delta x^6 = 0$, there is no solitonic interpretation of the D2-brane in the 4 dimensional theory. See figure 2(c). The D2-branes (macroscopically $\sim$ D-strings) can pass through each other classically, as the solution (2.2) shows. On the other hand, when $r \neq 0$, the theory acquires the fundamental field ψ and is in a Higgs phase, then the configuration (2.2) is not allowed on the moduli space. This classical fact is the origin of the difference between the reconnection property of colliding strings.

In the next section, we introduce appropriate coordinates to parametrize the moduli space (2.5) and solve the sigma model to show the inevitability of the reconnection of the colliding vortex strings. In section 4, we explain how a quantum mechanics on the classical D-string solution (2.2) will cause the reconnection.

[6]Though we have used a specific brane realization of the abelian Higgs model, there are many other brane configurations. However, any embedding of the model in string theory will reproduce the same result for the effective theory on the vortex string, after taking the decoupling limit (the low energy limit).

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3. Reconnection of colliding vortex strings

3.1 Effective field theory on vortex strings

The Moduli space metric on this Higgs branch defined by the D-term condition (2.5) was determined by a Kähler quotient technique in [18], which we shall utilize in the following. The D-term condition (2.5) can be solved by the parametrization [18]

$$Z = w\mathbb{1}_{2\times 2} + z\begin{pmatrix} 1 & \sqrt{2b/a} \\ 0 & -1 \end{pmatrix}, \qquad \psi = \sqrt{r}\begin{pmatrix} \sqrt{1-b} \\ \sqrt{1+b} \end{pmatrix} \tag{3.1}$$

where

$$a \equiv \frac{2|z|^2}{r}, \qquad b \equiv \frac{1}{a+\sqrt{1+a^2}}\,. \tag{3.2}$$

The parameter w describes the center-of-mass for the two vortex strings, while $2z$ parametrizes the relative position of them. This can be seen when real or imaginary part of the matrix Z is diagonalized. (These cannot be diagonalized simultaneously, though.) Using this parametrization, the metric on the moduli space spanned by z is given by [18]

$$ds^2 = g(|z|)dzd\overline{z}\,, \qquad g(|z|) \equiv \frac{|z|^2}{\sqrt{|z|^4 + r^2/4}}\,. \tag{3.3}$$

The center of mass position w is decoupled from z in this approximation of slow motion on the moduli space, so we concentrate on z. The effective action of the relative motion of the two vortex strings is then written as

$$S = \mathcal{T}\int dtdx\; g(|z|)\partial_\mu z(t,x)\partial^\mu \overline{z}(t,x)\,. \tag{3.4}$$

The approximation is valid if the lagrangian (the integrand in the above S) itself is small compared to 1. $\mathcal{T}$ is an analogue of the tension of strings, but this overall factor is irrelevant in the following classical computations.

The metric $g(|z|)$ describes a resolved cone geometry. This can be read from the asymptotic behavior of the metric,

$$g(|z|) \sim \begin{cases} \frac{2|z|^2}{r} & \text{for} \quad |z| < \sqrt{\frac{r}{2}} \\ 1 & \text{for} \quad |z| > \sqrt{\frac{r}{2}} \end{cases} \tag{3.5}$$

For large separation $|z| \gg \sqrt{r/2}$, the metric of course goes to the flat metric. But when the vortex strings are close ($z \sim 0$), we can make a coordinate transformation

$$\widetilde{z} \sim \frac{1}{\sqrt{2r}} z^2 \tag{3.6}$$

resulting in a metric $ds^2 \sim |d\widetilde{z}|^2$. That is, near the origin a well-behaving coordinate is $\widetilde{z}$ rather than z. As is obvious from this expression of the coordinate transformation, the correct interpretation of z is through the identification $z = \overline{z}$, that is, antipodal points in z space are identified [19]. This fact is in agreement with the observation by Ruback [20].

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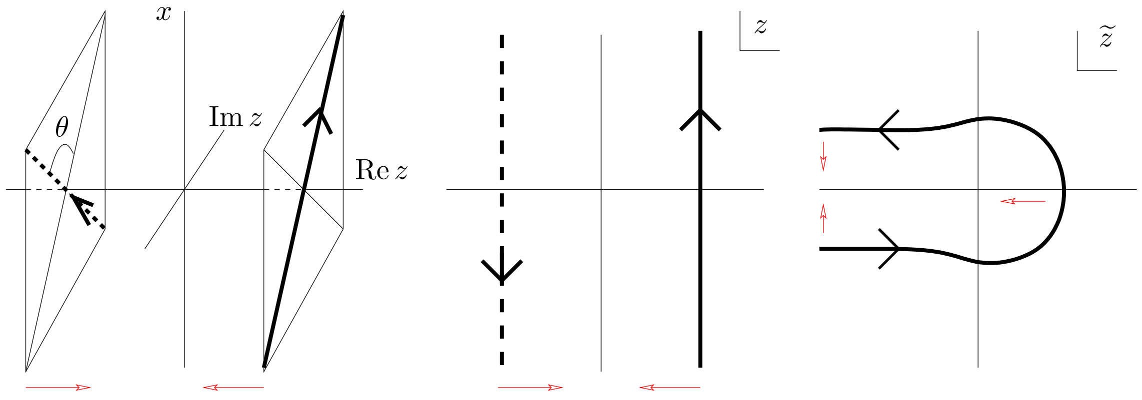

(a) (b) (c)

Figure 3: (a) Initial motion of the two vortex strings (the thick line and the dashed thick line with arrows indicating their orientations). Small arrows show the direction of the motion. (b) Projection of (a) onto the plane spanned by the complex coordinate z. The dashed line is a mirror partner of the thick line. (c) Configuration of (a)(b) mapped onto the plane spanned by $\widetilde{z}$.

Now let us consider two colliding gauge strings. We consider an initial condition at some time $t = t_{\rm ini}$ as

$$z = z_0 + i \tan\left(\frac{\theta}{2}\right) x \tag{3.7}$$

$$\dot{z} = -\frac{v}{2} \tag{3.8}$$

where the three parameters θ, z_0 and v are real and positive. The configuration of two vortex-strings is shown in figure 3(a). Immediately one can deduce that θ is the relative angle of the strings, and v is the relative velocity. Thus at $t = t_{\rm ini}$ the closest distance between the strings is $2z_0$. We assume that v and θ are small so that we can use the moduli space approximation. This actual string configuration in the 3 dimensional space spanned by x, z can be consistently projected onto the 2 dimensional plane spanned only by z, see figure 3(b). We may consider the relative motion of the strings on this projected complex z-plane.

Respecting the identification of the antipodal points, we make the following coordinate transformation in the target space:

$$\widetilde{z} \equiv \frac{z^2}{2(|z|^4 + r^2/4)^{1/4}}\,. \tag{3.9}$$

When $z \sim 0$, this transformation reproduces (3.6). This coordinate makes the geometrical picture of the metric clearer as follows. The geodesic distance in this coordinate $\widetilde{z} \equiv \rho e^{i\varphi}$ is written as

$$ds^2 = \left(1 + \left(\frac{df(\rho)}{d\rho}\right)^2\right) d\rho^2 + \rho^2 d\varphi^2 \tag{3.10}$$

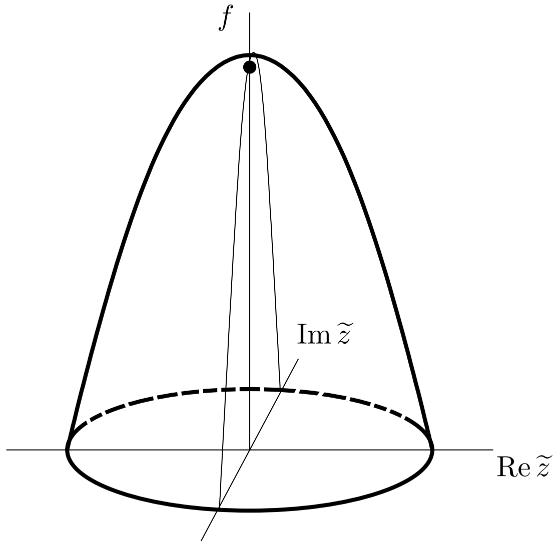

Figure 4: The moduli space metric is an induced metric on a 2 dimensional surface embedded in the 3 dimensional f-$\widetilde{z}$ space. This surface is a smoothed cone of a deficit angle π.

where $f(\rho)$ is a smooth function determined through (3.9). We need only its asymptotic values,

$$f(\rho) \sim -\sqrt{3}\rho \qquad (\rho \sim \infty)\,, \qquad f(\rho) \sim -\sqrt{\frac{2}{r}}\rho^2 \qquad (\rho \sim 0)\,. \tag{3.11}$$

The expression (3.10) shows that it describes a smeared surface of a cone with a deficit angle π. It is a metric induced on a 2 dimensional hypersurface in 3 dimensions spanned by $\widetilde{z} = \rho e^{i\varphi}$ and a hypothetical new coordinate f, with the embedding $f = f(\rho)$. The deficit angle π can be read from the asymptotic slope of f, $\lim_{\rho\to\infty} |df/d\rho| = \sqrt{3}$. See figure 4.

3.2 Proof of reconnection of colliding vortex strings

Note that for scattering of vortices (not the vortex strings) in 2 spatial dimensions, the moduli space metric is the same as (3.10). The initial condition of the colliding vortices is (3.8) with $\theta = 0$: this is a simple dimensional reduction along x from the vortex string case. And it is obvious that the worldline trajectory of the vortex is just a straight line going through the top of the cone — a straight line in the $\widetilde{z}$-space. In terms of z, this means a right-angle scattering [20, 19].

The motion of the colliding vortex strings is described by a Polyakov string whose motion is constrained on this smeared cone with a particular initial motion. The initial condition (3.8) is given as figure 3(c) in the $\widetilde{z}$-plane, and if we map it onto the cone, it is a Polyakov string winding the top of the smeared cone, see figure 5(a). The Polyakov string moves slowly toward the top of the hill, and because the top is smeared the string can smoothly travel beyond the top and come down, as in figure 5(b), which we call a final configuration. On the projected $\widetilde{z}$-plane, the shape of the final configuration is shown in figure 6(c). Mapping this back to the original 3 dimensional space, we obtain figure 6(a),

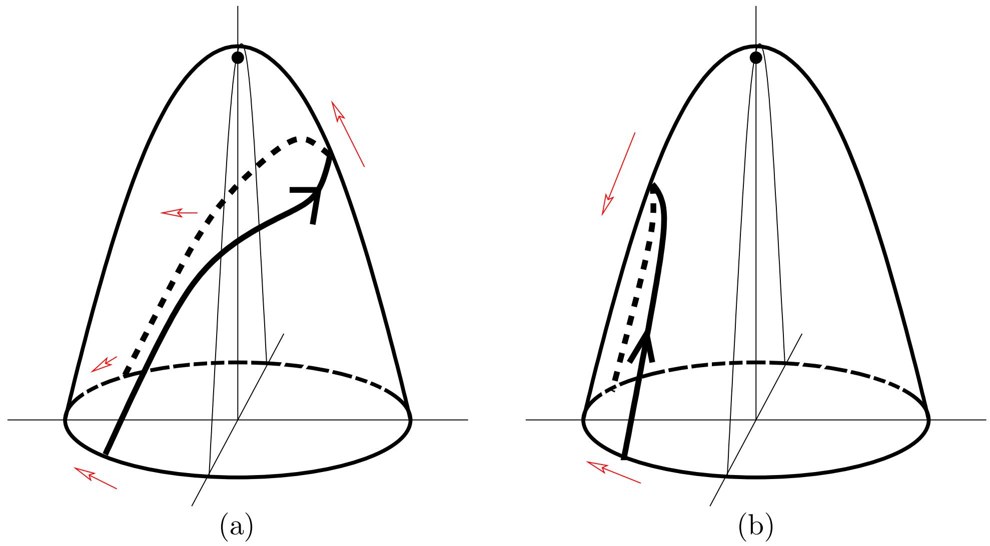

Figure 5: (a) The initial configuration of the Polyakov string (the thick line with an arrow) on the cone corresponding to two straight vortex strings colliding. The small arrows show the direction of the motion of the Polyakov string. (b) The Polyakov string travels without feeling any singularity through the top of the cone, and arrives at the final configuration.

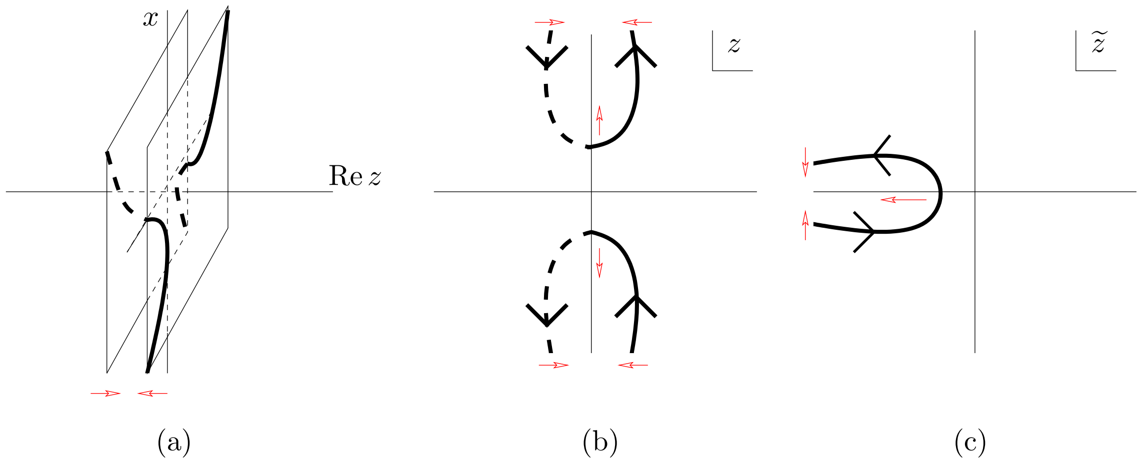

Figure 6: (c) The string configuration in $\widetilde{z}$ space after a while. (b) The configuration of (c) is mapped to the space z. (a) It is lifted back to the original 3 dimensional space. The vortex strings are reconnected. The thick line is now connected with its original mirror string (dashed thick line).

which shows that the vortex strings are reconnected. We saw here that a smooth travel of the Polyakov string on the smeared cone geometry turns out to give the reconnection of the original colliding vortex strings. This proof is valid as long as the sigma model description (3.4) does not break down. Thus, the reconnection always occurs for small collision velocity $v \ll 1$ and small intersection angle $\theta \ll 1$.

If we take a slice $x = 0$ of the vortex strings, it can be interpreted as a scattering of a pair of vortices in 2 dimensions. For any slice of a fixed value of x, the reconnection

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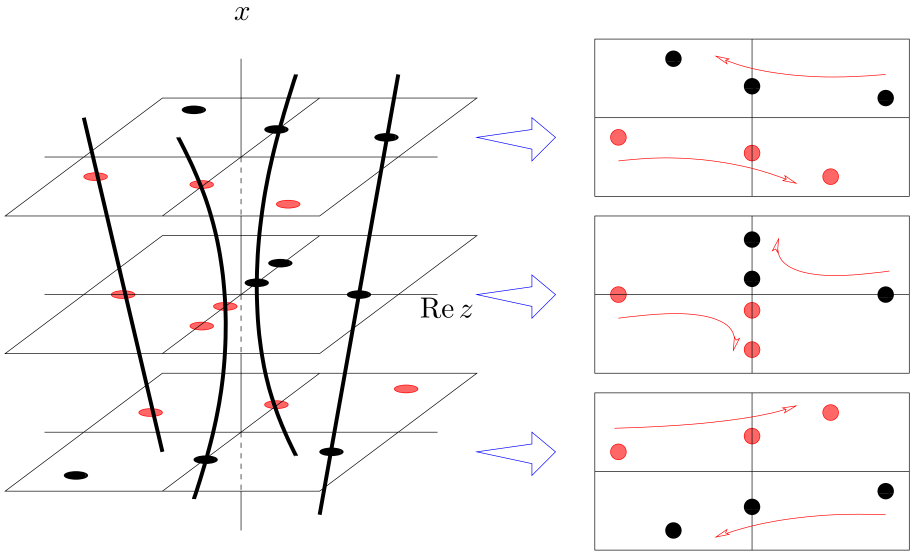


Figure 7: Reconnection of the colliding strings (Left) can be understood as a collection of colliding vortices in 2 dimensions with various impact parameters (Right). At the slice $x = 0$, the vortex string reconnection is equivalent to the right-angle scattering of the vortices.

of the colliding vortex strings can be seen as a scattering of the vortices with various impact parameters given by (3.8) with the value of x substituted. This is the intuitive understanding of the reconnection, see figure 7. The slice $x = 0$ has been considered first in ref. [13].

When the collision velocity is large enough, numerically the velocity upper bound has been observed for the reconnection to occur [6]. In this situation, the sigma model approximation becomes incorrect, but in the next subsection we derive the upper bound analytically by looking at a relativistic consistency condition for the shape of the reconnected vortex strings. But before closing this subsection, we would like to give an intuitive picture of the existence of the upper bound, using the figure of the cone, figure 4, 5. Suppose that even for vortex strings moving relativistically fast, the cone geometry captures a correct dynamics. When the relative velocity is too fast, the motion of the Polyakov string at the bottom of the cone in the figures is very fast, and it goes around the cone faster than the string passes the top of the cone. See figure 8(a). Then the final configuration is not in the region Re $\widetilde{z} < 0$ as in figure 5(b) but comes back to the region Re $\widetilde{z} > 0$ (figure 8(b)). A careful look at the string reveals that the orientation of the final Polyakov string is opposite compared to the initial configuration of figure 5(a). This means that in the original spacetime the vortex strings pass through each other without reconnection.

3.3 Static vortex strings: reconnection via pair annihilation

We have seen that the map to the Polyakov string on the cone geometry provides a physical and clear understanding of the reconnection of the colliding vortex strings. It would be better to confirm the dynamics by explicitly solving the equations of motion of the sigma model action (3.10) with the given initial condition (3.8), but it turns out to be complicated.

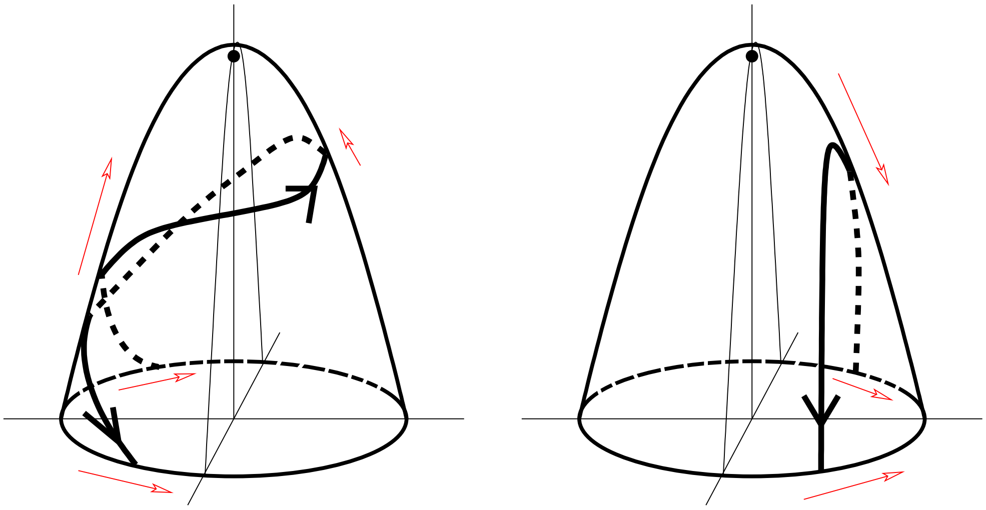


Figure 8: (a) When the relative velocity is large, the encircling speed of the Polyakov string far from the tip of the cone (the speed at the bottom in the figure) is so large that the string self-intersects. (b) As a consequence, after a while the Polyakov string is brought on the side of positive $\mathrm{Re}\,\tilde{z}$, but the orientation is opposite to the original configuration in figure 5 (a).

In this subsection we solve them with a static ansatz, which provides an evidence supporting the inevitability of the reconnection for vortex strings. The static ansatz can be understood as an adiabatic collision of the vortex strings, that is, an infinitesimally small collision velocity v. We will see that static configurations of a pair of the vortex strings tend to be aligned in orientations opposite to each other: locally around the intersection point it becomes a string anti-string pair. Field theoretically the pair of the vortex string and the anti vortex string should be unstable and would decay to the vacuum (only around the intersection), then the resulting configuration should be the reconnected vortex strings. Our result is coincident with [21] in which a different effective analysis on the vortex strings has been performed.

The equations of motion for the field $z(t,x) \equiv z_1 + iz_2$ of the sigma model (3.4) in two dimensions are

$$\partial_\mu \left(g(|z|)\partial^\mu z\right) = 0\,. \tag{3.12}$$

The static ansatz $\partial_0 z = 0$ can be consistently imposed. Furthermore, we restrict our attention to the case where $\mathrm{Re}z = z_1$ is a real constant z_0. This means that one vortex string lies on a two dimensional plane $\mathrm{Re}\,z = z_0$ while the other lies on $\mathrm{Re}\,z = -z_0$. Because $\partial z_1 = 0$, this restriction is again consistent with the equations of motion, thus the dynamical variable to be solved is only $z_2(x)$. The equation of motion in this case can be integrated immediately to give

$$x = C\int_0^{z_2} g\, dy \tag{3.13}$$

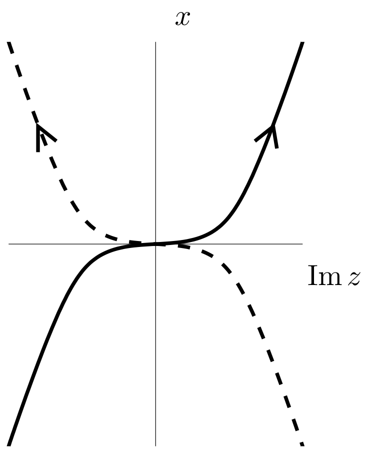


Figure 9: A static solution of two vortex strings lying on the same plane $\mathrm{Re}\, z = 0$. The dashed line is the mirror string (specified by $-z$). At the origin, the orientation of the solid line is opposite to that of the dashed line, showing the tendency to form a pair of string and anti-string.

where the argument of g is given by $|z| = \sqrt{z_0^2 + y^2}$, and thus

$$g = \frac{z_0^2 + y^2}{\sqrt{(z_0^2 + y^2)^2 + r^2/4}} \,. \tag{3.14}$$

We have fixed an integration constant by putting $z_2(x = 0) = 0$ without losing generality, and C is another integration constant which can be determined as follows. The relative angle between the vortex strings can be seen in their asymptotic slopes. Noting that above g goes to the unity for large y, we find $x \sim C z_2$ for large z_2. Therefore

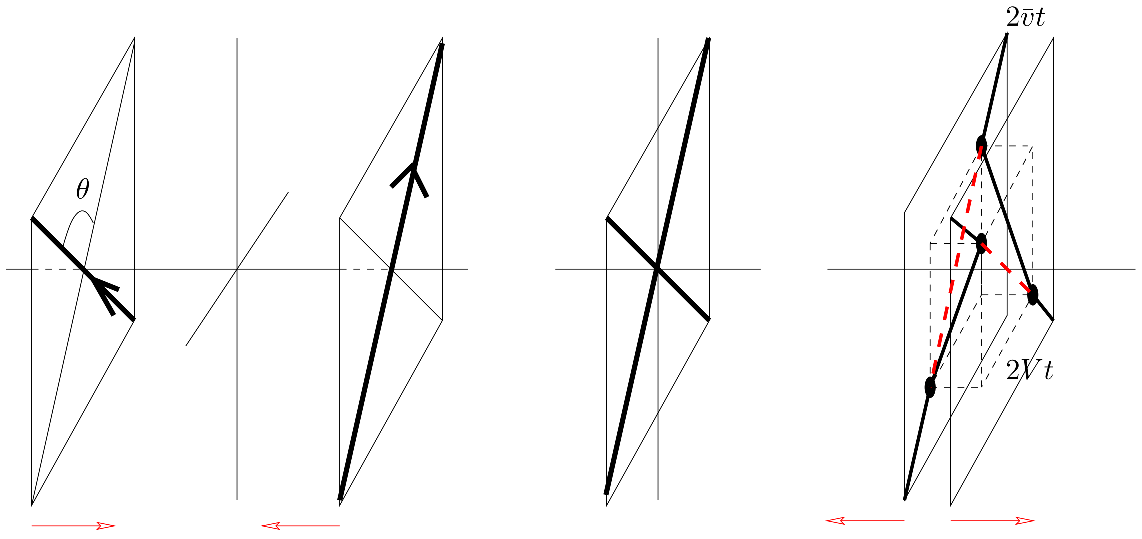
$$C = \frac{1}{\tan(\theta/2)} \,. \tag{3.15}$$

Because the function g is a monotonically increasing function of z_2 for fixed $z_1 = z_0$, we deduce that $\partial x/\partial z_2$ is also monotonically increasing and approaching $1/\tan(\theta/2)$ asymptotically. The value of $\partial x/\partial z_2$ at the origin $x = z_2 = 0$ is

$$\left.\frac{\partial x}{\partial z_2}\right|_{x=z_2=0} = \frac{1}{\tan(\theta/2)} \frac{z_0^2}{\sqrt{z_0^4 + r^2/4}} \,. \tag{3.16}$$

The important point is that this vanishes for $z_0 = 0$. See figure 9. For the value $z_0 = 0$, two vortex strings are on the same two dimensional plane $z_1 = 0$. At the origin $z_1 = z_2 = x = 0$ they intersect, and in fact become anti-parallel to each other: a pair of a vortex string and an anti vortex string. They should annihilate with each other, and the resulting configuration should be the reconnected vortex strings. The reasoning that the local pair annihilation of strings leads to the reconnection is the same as what was found in [16] in D-brane reconnection.

The fact that two static vortex strings tend to intersect with relatively opposite orientation is a supporting evidence for the inevitability of the reconnection of the colliding strings. Our result is consistent with ref. [21] in which a different effective description of the vortex strings was adopted.



(a) (b) (c)

Figure 10: Schematic figure of reconnection of vortex strings. (a) Two vortex strings (thick lines) are colliding. Small arrows indicate the directions of the string motion. (b) They collide at $t = 0$. (c) They reconnect with each other. The shape after the reconnection is assumed in such a way that the reconnected part is on a dashed box which expands. The blobs represent kink points. The thick dashed lines are the strings which would have been present if the reconnection didn't occur.

3.4 Velocity upper bound for reconnection of vortex strings

The numerical simulations [6] indicate that there is an upper bound for the relative velocity, for the vortex strings to be reconnected. The "probability" of reconnection, termed in [6] should be understood as a condition on the velocity for the reconnection to occur classically. For vortex strings, the reconnection is a purely classical phenomenon. There is a paper [13] which tried to derive in field theories this upper bound by analytic calculations concerning deformation of classical solutions of two coincident vortices with some ansatz. The result is the following velocity upper bound,

$$\bar{v} < \sqrt{\frac{4\alpha(1-\cos\theta)}{1+4\alpha(1-\cos\theta)}}\,, \tag{3.17}$$

where $\bar{v}$ is the velocity of the colliding vortex strings in the center-of-mass frame (and thus related to the relative velocity v as $v = 2\bar{v}/(1+\bar{v}^2)$), and α is an unknown parameter introduced in the ansatz in [13]. This dependence on θ coincides with the numerical results in [6] qualitatively. In this section we derive this velocity upper bound (3.17) by a geometrical consideration, without referring to any classical field configuration. Thus, although the condition (3.17) has been derived with a specific field configuration in a certain theory, we show that the same upper bound holds true for general vortex strings.

First we assume that the shape of the reconnected vortex strings is as in figure 10, and neglect the effect of the energy of the kink points. In the center-of-mass frame, the velocity of the original strings is $\bar{v}$ and the velocity of the reconnected region is V. Generically V

may depend on time but here we consider the configuration at times just a little after the collision and so t is close enough to the collision incidence $t = 0$ and thus we may regard V as a constant. Note that the direction of the motion V is perpendicular to that of $\bar{v}$, due to our assumption on the shape of the strings.

Let us consider the energy gain by assuming that the reconnection has occurred,

$$\delta E = E_+ - E_- \,. \tag{3.18}$$

Here E_+ is the energy produced by the reconnection, that is, the energy of the string between the generated kinks. (In figure 10, this is the energy of the solid lines along the box surface.) E_- is the energy of the original strings which disappeared after the reconnection. (In figure 10, this is the energy of the dashed lines.) Since the location of the kink points are

$$\left(Vt\cot\left(\frac{\theta}{2}\right), Vt, \bar{v}t\right), \qquad \left(-Vt\cot\left(\frac{\theta}{2}\right), Vt, -\bar{v}t\right), \tag{3.19}$$

an explicit calculation gives

$$E_+ = 4\mathcal{T}\frac{t\sqrt{\bar{v}^2 + V^2\cot^2(\theta/2)}}{\sqrt{1-V^2}} \,. \tag{3.20}$$

The denominator comes from the gamma factor for relativistic motion of the strings. The numerator is the length of the reconnected strings. On the other hand, the loss of the energy E_- is

$$E_- = 4\mathcal{T}\frac{Vt}{\sqrt{1-\bar{v}^2}\sin(\theta/2)} \,. \tag{3.21}$$

Thus we obtain the total energy gain,

$$\delta E = 4\mathcal{T}t\left(\frac{\sqrt{\bar{v}^2 + V^2\cot^2(\theta/2)}}{\sqrt{1-V^2}} - \frac{V}{\sqrt{1-\bar{v}^2}\sin(\theta/2)}\right). \tag{3.22}$$

Given an initial velocity $\bar{v}$, if this energy gain can be negative for an appropriately chosen velocity V, then the reconnection should occur.

We note here that a-priori there is a restriction on the value of V, because the kink points cannot propagate faster than the speed of light. This condition can be easily read from the kink coordinates (3.19), giving a constraint

$$0 \le V \le \sqrt{1-\bar{v}^2}\sin(\theta/2) \; (\equiv V_{\max}) \,. \tag{3.23}$$

At the allowed velocity minimum $V = 0$, the energy gain δE is positive, while at the allowed velocity maximum $V = V_{\max}$, the energy gain vanishes (see figure 11). Checking the sign of the derivative $\partial(\delta E)/\partial V$, we find the condition for $\bar{v}$ to provide a negative δE with a suitably chosen V in the allowed region (3.23), as

$$\bar{v} < \frac{\sin(\theta/2)}{\sqrt{1+\sin^2(\theta/2)}} \,. \tag{3.24}$$

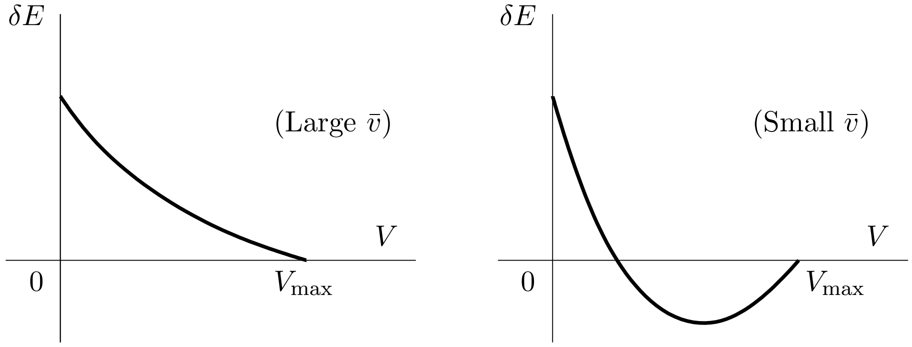


Figure 11: The energy gain in the allowed region of V. For large $\bar{v}$, there is no value of V in the allowed region $0 < V < V_{\rm max}$ which gives a negative δE (Left), while for small $\bar{v}$ there exist such values (Right).

Thus we have an upper bound for $\bar{v}$ to have the reconnection. This bound is derived only from a geometrical consideration, but surprisingly, it is the same as (3.17) found in [13], if we choose the parameter $\alpha = 1/8$.[7]

4. Reconnection of colliding D-strings

In this section we give an estimation of the probability of reconnection of two D-strings when they collide with a constant relative velocity, by using a low energy effective field theory on D-strings (2.1). As shown in section 2, there is a classical solution of colliding D-strings passing through each other without reconnection. The reconnection is triggered by *quantum tachyonic fluctuations* around the classical solution, as we will see. Thus the crucial point is that the reconnection is actually a probabilistic event, in contrast to the case of the classical reconnection of the colliding vortex strings studied in section 3. In the following, we compute the reconnection probability of the colliding D-strings whose classical trajectory is given by (2.2). We shall compare our result with the calculation by the string worldsheet theory given in [9] and find a qualitative agreement.

4.1 Tachyon condensation and reconnection

First let us briefly review how the reconnection of intersecting D-strings is realized by the tachyon condensation in terms of Yang-Mills theory on the D-strings [16]. For intersecting D-branes, there appear tachyonic fluctuations around the intersection point, and the condensation of them shows a reconnection. This has been explicitly demonstrated in [16], and further studied in [22−24].[8]

We start with the D-string action (2.1) which is the 1+1 dimensional SU(2) Yang-Mills theory. Precisely, this is a low-energy effective action of two coincident parallel D-strings.

[7] In the present evaluation of the velocity upper bound, we assumed the shape of the reconnected vortex strings and the position of the kinks. Thus it is possible that the true upper bound may be a little bigger than what we have obtained here, (3.24).

[8] For related earlier references, see [15, 25].

Following [16], we turn on an intersection angle θ (while keeping the D-strings on a two-dimensional plane, $\bar{v} = 0$ in (2.2) and thus $\Phi_3 = 0$) and perform a fluctuation analysis, to get the lowest fluctuation mode

$$\Phi_2 = \frac{T(t)}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \exp\left[-\frac{\tan(\theta/2)}{2\pi l_s^2} x^2\right], \tag{4.1}$$

$$A_x = \frac{T(t)}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \exp\left[-\frac{\tan(\theta/2)}{2\pi l_s^2} x^2\right]. \tag{4.2}$$

Here x is the coordinate along the original D-string worldvolume, and we took the gauge $A_0 = 0$. This fluctuation eigen-mode has the mass squared

$$m^2 = -\frac{\tan(\theta/2)}{\pi l_s^2} + \frac{(2z_0)^2}{(2\pi l_s^2)^2} \tag{4.3}$$

with $z_0 = 0$. The parameter z_0 appearing here becomes non-zero if another transverse separation along Φ_3 between the D-strings is turned on as $2\pi l_s^2 \Phi_3 = \mathrm{diag}(z_0, -z_0)$. (This z_0 has been set to zero in [16].) Inclusion of nonzero constant z_0 in the fluctuation analysis is straightforward, and the result (4.3) coincides with the string worldsheet analysis [14] for $\theta \ll 1$. In addition, there are infinite number of other eigen-modes with higher mass squared, which reproduce a part of the worldsheet spectra of a string connecting the two D-strings [16, 23, 26, 27].

Knowing the eigen-mode decomposition along the direction x, we can reduce the system by one dimension lower, by integrating over x, to obtain an effective action for the coefficient function $T(t)$ in (4.2). This coefficient function is the tachyon field, because when the separation $2z_0$ is small enough, the mass squared (4.3) becomes negative. Substituting the profile (4.2) back to the Yang-Mills action (2.1), we obtain

$$\begin{aligned} S &= \frac{1}{2}\mathcal{T}_{\mathrm{D1}}(2\pi l_s^2)^2 \int dt dx \left[(\partial_t T(t))^2 - m^2 T^2\right] \exp\left[-\frac{2\tan(\theta/2)}{2\pi l_s^2} x^2\right] \\ &= \frac{1}{g_T} \int dt \left[\frac{1}{2}(\partial_t T(t))^2 - \frac{1}{2} m^2 T^2\right]. \end{aligned} \tag{4.4}$$

This is the tachyon action in one dimension which we use in this section. The overall coefficient is related to the original parameters as

$$g_T = \frac{g_s \sqrt{\theta}}{2\sqrt{2}\pi^2 l_s^3}. \tag{4.5}$$

We may regard the action (4.4) as a quantum mechanics of a particle moving in a harmonic oscillator: in this interpretation, the value T is the position of a particle, and the overall factor $1/g_T$ is the mass of the particle.

When the tachyon acquires a vev, this immediately leads to the reconnection. The eigen-mode T enters in the off-diagonal entries of the matrix Φ as in (4.2) and thus, diagonalizing the matrix Higgs field Φ_2, we obtain the transverse displacement of the D-strings (for $z=0$) as

$$2\pi l_s^2\phi = \pm\sqrt{(\tan(\theta/2))^2x^2 + (\pi l_s^2)^2\exp\left[-\frac{\tan(\theta/2)}{\pi l_s^2}x^2\right]T^2}\,. \tag{4.6}$$

Therefore, the tachyon vev is related to the separation of the reconnected D-strings Δ (the closest points on the D-strings are given by $x=0$) as

$$\Delta = 2\pi l_s^2 T\,. \tag{4.7}$$

We assume that this relation holds also for $z \neq 0$ in the following analysis of D-string collision.[9]

4.2 Time evolution of tachyon wave function and reconnection condition

The tachyon mass squared m^2 is negative when the D-strings are close enough to each other, but otherwise, it is positive and gives a usual harmonic oscillator. Thus the collision of the angled D-strings provides time-dependent transitions between these two situations. To describe the collision of the angled D-strings, we turn on a small relative velocity $v(>0)$,

$$2z_0 = vt\,, \tag{4.8}$$

as in (2.2). For very small velocity, the fluctuation analysis in the previous subsection is valid ($v \ll \theta \ll 1$). The time when the tachyon mass squared becomes zero is

$$t = \pm t_0\,, \qquad t_0 = \frac{l_s\sqrt{2\pi\theta}}{v}\,. \tag{4.9}$$

The one-dimensional action (4.4) is quite simple, a harmonic oscillator with a time-dependent frequency. The frequency becomes imaginary for the period

$$-t_0 < t < t_0 \tag{4.10}$$

during which the harmonic potential becomes upside down and there exists a tachyonic instability.

The physical picture of quantum evolution of the system is as follows.[10] For large negative t, the system is with the harmonic potential and so the wave function $T(t)$ is a gaussian function whose width is determined by the frequency changing slowly in time. Then at $t=-t_0$, the system drastically changes to an unbounded one and the wave function begins to spread out very quickly. However, when $t=t_0$, the potential comes back to the original harmonic form and the system recovers to be a bounded well-defined system. This means that the wave function rapidly spreads out with the unbounded potential only for

[9] For non-zero z_0, the two Higgs fields Φ_2 and Φ_3 are not simultaneously diagonalizable and thus this geometrical interpretation is not rigorous.

[10] A quantum analysis of the reconnection was studied in [28].

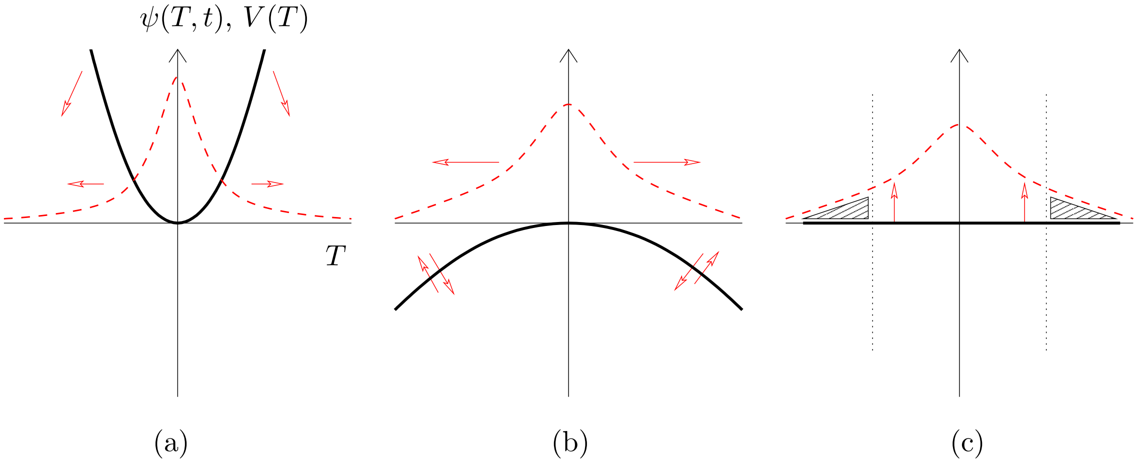


Figure 12: Time evolution of the harmonic potential (thick lines) and the tachyon wave function (dashed lines). The small arrows denote their time evolution. (a) When the D-strings are far enough from each other ($t < -t_0$), the tachyon mass squared is still positive and the wave function is a gaussian. (b) For $-t_0 < t < t_0$, the potential becomes inverse-harmonic, and the wave function expands rapidly. (c) At $t = t_0$, the system gets back to the bounded harmonic potential. For large tachyon values the reconnection will occur. Thus the reconnection probability is evaluated by integrating the shaded region of the wave function.

the finite period, $-t_0 < 0 < t_0$. See figure 12. At the end of the unstable era, $t = t_0$, we have a well-spread wave function which allows a large value of $T(t)$ and gives the reconnection. Then how can we know if the reconnection occurs or not? We introduce the following geometrical requirement for the tachyon field to be observed to give the reconnection. At the instant $t = t_0$, two D-strings will be separated (asymptotically) by vt_0 in the Φ_3 direction. Thus, as seen in figure 13, the reconnected distance Δ along Φ_2 should be larger than this vt_0 so that the reconnection provides energy reduction. The geometrical requirement for the reconnection to be observed at $t = t_0$ should be

$$vt_0 < \Delta\,. \tag{4.11}$$

This is translated to the condition for the tachyon field, using (4.7) and (4.9) as

$$T > \frac{\sqrt{\theta}}{\sqrt{2\pi} l_s}\,. \tag{4.12}$$

Therefore, to compute the reconnection probability for the colliding D-strings, first we compute the time evolution of the tachyon wave function, and at $t = t_0$ we evaluate the probability to have T satisfying the condition (4.12). We will perform this in the next subsection explicitly.

4.3 Reconnection probability

Let us consider the evolution of the tachyon wave function $\psi(T,t)$. Since it is technically difficult to get the exact expression of this wave function for the time-dependent frequency,

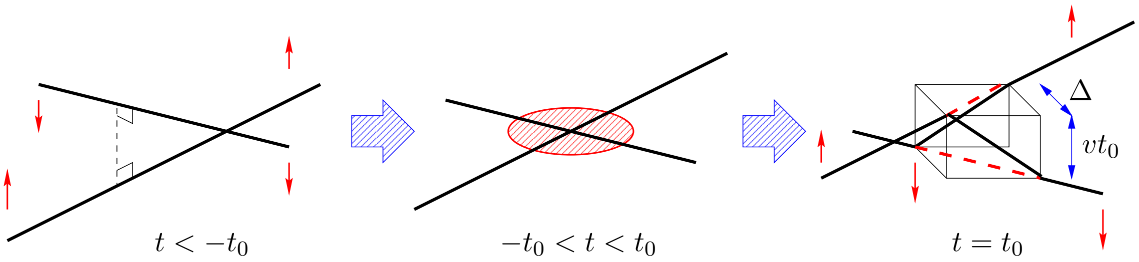


Figure 13: Three stages for the reconnection of the D-strings. Two D-strings are approaching (Left), and tachyon wave function spreads (Center). At last, the D-strings are reconnected (Right). The dashed lines indicate the case where D-strings were not reconnected. Essentially this figure is the same as figure 10, but just pushed sideways to show the reconnected region explicitly.

we adopt the following crude approximation for the time-dependence of the frequency:

$$m^2 = \frac{(vt)^2}{(2\pi l_s^2)^2} \qquad (t < -t_0) \tag{4.13}$$

$$m^2 = -m_{\text{second}}^2 \equiv -s\frac{\theta}{2\pi l_s^2} \;(-t_0 < t < t_0) \tag{4.14}$$

In the first period, the mass squared is positive and proportional to t^2, where we neglected the first term in (4.3). This approximation is valid if $t \ll -t_0$, but we are going to use this m^2 until $t = -t_0$. In the second tachyonic period, we approximate the mass squared by its typical tachyonic value, and s is an $\mathcal{O}(1)$ parameter. One may choose $s = 2/3$ which is the average tachyon mass squared in this second period. This approximation of the second period is especially useful since we can apply the results obtained in [29], where time evolution of a gaussian wave function in the tachyonic harmonic potential was calculated.

It is easy to show that the technique developed in [29] can be used also for non-constant mass squared such as the one in the first period (4.13). The wave function is of the form

$$\psi(T,t) = A(t)\exp\left[-B(t)T^2\right]. \tag{4.15}$$

Solving the equation for $B(t)$ in $1/t$ expansion in the first period (4.13), one obtains

$$B(t) = -\frac{v}{4\pi l_s^2 g_T}t + \frac{i}{4g_T}\frac{1}{t} + \cdots. \tag{4.16}$$

Thus at $t = -t_0$, the initial condition for the second period is given by

$$B(t=-t_0) = \frac{v}{4\pi l_s^2 g_T}t_0 - \frac{i}{4g_T}\frac{1}{t_0}. \tag{4.17}$$

In the second period, the evolution of the wave function was determined in [29]. The solution is

$$B(t^{\text{GP}}) = \frac{1}{2a^2}\tan(\phi - i\omega t^{\text{GP}}) \tag{4.18}$$

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where ϕ is a parameter in the solution, and in correspondence to our language,

$$a^2 = \frac{g_T}{m_{\rm second}}\,, \qquad \omega = m_{\rm second}\,. \tag{4.19}$$

$t^{\rm GP}$ is the time in the convention of [29] whose zero $t_0^{\rm GP}$ is different from that of our time as $t^{\rm GP} - t_0^{\rm GP} = t + t_0$. We may derive this $t_0^{\rm GP}$ by comparing (4.17) and (4.18) at $t^{\rm GP} = t_0^{\rm GP}$. We obtain two equations

$$\frac{\sin 2\phi}{\cos 2\phi + \cosh 2\omega t_0^{\rm GP}} = \sqrt{\frac{\theta}{s}}\,, \qquad \frac{\sin 2\phi}{\sinh 2\omega t_0^{\rm GP}} = \frac{2\theta}{v}\,. \tag{4.20}$$

For small θ and v/θ, these can be solved to give

$$\sin 2\phi \simeq 2\sqrt{\frac{\theta}{s}}\,, \qquad \omega t_0^{\rm GP} \simeq \sqrt{\frac{v\theta}{s}}\,, \tag{4.21}$$

which gives quite small $t_0^{\rm GP}$ ($\ll |t_0|$), thus we may use a relation

$$t^{\rm GP} \simeq t + t_0\,. \tag{4.22}$$

For large ωt, a simple expression for the wave function was given in [29],

$$|\psi|^2 = \sqrt{\frac{2\sin 2\phi}{a^2\pi}}\exp\left[-\omega t^{\rm GP} - 2(\sin 2\phi)e^{-2\omega t^{\rm GP}}\frac{T^2}{a^2}\right]. \tag{4.23}$$

The wave function is still gaussian, due to which it is easy to finally evaluate the probability of having (4.12), $T > \sqrt{\theta}/\sqrt{2\pi}l_s$ at $t = t_0$: the reconnection probability is

$$P = 2\int_{\sqrt{\theta}/\sqrt{2\pi}l_s}^{\infty} dT\; |\psi(T, t^{\rm GP} = 2t_0)|^2\,. \tag{4.24}$$

This is basically an error function whose asymptotic form is

$$\int_u^\infty ds\; e^{-s^2} = \frac{e^{-u^2}}{2u} + \cdots\,, \tag{4.25}$$

thus we obtain

$$P \simeq \frac{\sqrt{g_s}}{2\pi^{3/4}\theta^{3/4}}e^{2\sqrt{s}\theta/v}\exp\left[-\frac{4\sqrt{\pi}\theta^{3/2}}{g_s}e^{-4\sqrt{s}\theta/v}\right]. \tag{4.26}$$

This is the probability of the reconnection to occur for colliding D-strings.

Surprisingly, the result is very close to that of [9], a string worldsheet calculation,

$$P = \exp\left[\left(4 - \frac{v}{2g_s}\right)e^{-\pi\theta/v}\right]. \tag{4.27}$$

In fact, both have the "nonperturbative" factor in the exponent, $-e^{-\theta/v}/g_s$. This is quite interesting since the derivation of these results are very different. The result (4.27) [9] has been derived by evaluating the probability of creation of pairs of strings and anti-strings at the moment of the collision of the D-strings, and looking at the force balance of the created string junctions. Our result (4.26) is based on the effective field theory on the D-strings and the tachyon condensation.

As noted in [9], when the string coupling g_s becomes small, the reconnection probability gets very small. Intuitively this is obvious in our derivation, since small $g_{\rm s}$ corresponds to a large $1/g$, that is, a particle with a large mass in the quantum mechanics when T is identified with the position of the particle. Quantum evolution of a heavy particle is slow, and thus in our language correspondingly, the heaviness of the D-strings is reflected in the final form of the reconnection probability (4.26).

A slight discrepancy between (4.26) and (4.27) is found in the exponent, the coefficient of the nonperturbative factor is $\theta^{3/2}$ in our case while v in (4.27). This difference might originate in the following point: in [9] the reconnection condition is derived by a requirement that kinks generated by the creation of the F-strings between the D-strings cannot propagate faster than the speed of light. In our case, the lower-bound for the tachyon vev was given by the information of the geometrical shape of the reconnected D-strings. These might be related to each other in a more precise treatment of the quantum mechanics, since we have adopted a crude approximation (4.13), (4.14) for the time dependence of the potential which should have been dependent on the velocity v. Furthermore, we have assumed that the tachyon vev is related directly only to the horizontal separation Δ, which can be improved. Another missing factor 4 present in (4.27) might appear in our calculation if we take into account fermions in this system, since actually this 4 came into the calculation in [9] from worldsheet fermions.

5. Summary and Discussions

The intrinsic difference on collisions of the vortex strings in the abelian Higgs model and the D-strings in superstring theory is that the reconnection occurs classically for the former while for the latter it is merely a probabilistic event. This difference may be crucial for recognizing if cosmic strings in future observations/early universe scenarios are actually D-strings or not. In this paper we have clarified theoretically the origin of this difference, by studying the effective field theories on the vortex strings and on the D-strings. For the vortex strings, the theory is a sigma model whose target space is a smeared cone geometry, and a Polyakov string moving smoothly in this geometry describes the classical reconnection, showing its inevitability. On the other hand, the D-string action allows a classical solution of colliding D-strings passing without the reconnection, and we performed a quantum time evolution of the tachyon wave function to give the reconnection probability. The reconnection is in fact the tachyon condensation.

Then, why are the classical behaviors so different for these two strings? Intuitively, it seems that the reconnection should always occur for collision of any slowly moving strings, because the reconnection decreases the total energy which is given by the tension multiplied by the length of the strings. This should be a classical understanding of the reconnection, and looks to be consistent with the result for the vortex strings which always reconnect. Then why the D-strings do not reconnect classically? A possible answer to this question

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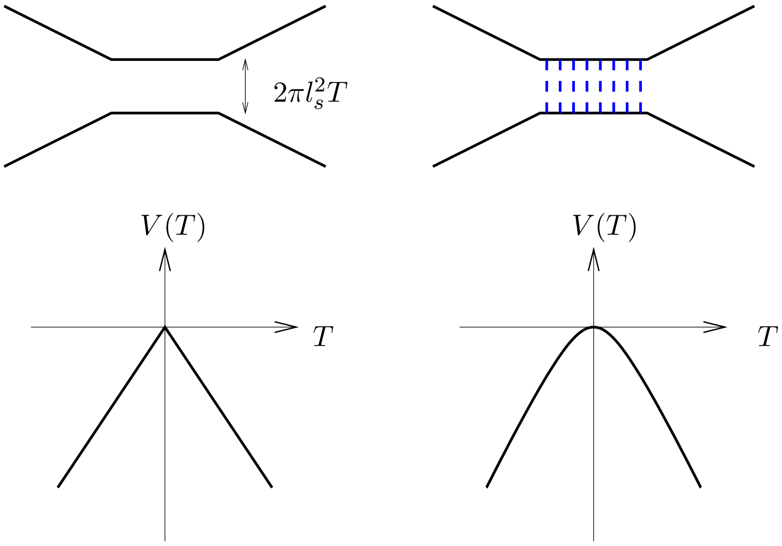

Figure 14: A schematic picture of the bond connecting the reconnected D-strings [23]. The upper-left figure shows strings reconnected hypothetically without the bond, while in the upper-right figure the bond is created in the actual reconnection of D-strings. If the bond was not created, the total energy of the D-strings would have exhibited a non-zero one-point function at the zero separation (lower-left). This is not the case, because we have a tachyonic fluctuation which can be defined only if the top of the potential is a smooth maximum (lower-right). The difference in potential between the two graphs is supplemented by the existence of the bond energy.

is found in [23] where the energy of the created "bond" connecting the reconnected D-strings is evaluated.[11] This "bond" whose presence was suggested in [28, 22] should be there so that the classical reconnection is prevented. The reconnection of D-strings is always accompanied by the creation of the bonds, and resultantly, the total energy does not decrease for infinitesimal reconnections [23]. See figure 14.

Concerning the difference between the vortex strings and the D-strings, the following question[12] is worth answered: according to Sen's conjecture, the D-strings are vortex-like topological defect in tachyon condensation on a brane-antibrane system. The action of the system is given by the tachyon field and gauge fields, and so it is almost the same as the abelian Higgs model. Thus any D-string can be regarded as a vortex string in this sense. Then, why can those be different?[13] Our answer to this question is that actually the action of the brane-antibrane is *not* the abelian Higgs model, because there exists infinite number of massive fields whose mass scale is the same as that of the tachyon field. Only when one considers all the tower of the excitations simultaneously, Sen's descent relation is properly realized: a D-string remains as a topological defect while the brane-antibrane

[11]The identity of the bond might be a bunch of fundamental strings, but classically the bond has no charge and thus is difficult to identify. But it should be something related to the tachyon matter [30] because the reconnection is the tachyon condensation [23].

[12]A related question was given in [31].

[13]Concerning this question, it would be interesting to pursue the connection proposed in [32].

disappears. In fact, the disappearance of the brane-antibrane can be shown only if one employs string field theories. Extracting just the tachyon field and the gauge field does not provide the D-string correctly. This is the very difference between ordinary field theories such as the abelian Higgs model and the string field theories. In ordinary field theories, any condensation of a field does not modify the number of physical degrees of freedom, while in string field theories this occurs, which is the significance of Sen's conjectures.[14]

In section 3, we have parametrized the Moduli space (2.5) by the coordinates (3.1) in which z specifies the location of the vortex strings. As mentioned in the footnote there, this z is merely the approximate location of the strings because the real part of the matrix Z and the imaginary part are not simultaneously diagonalizable for generic z. This is reflected in the D-term condition (2.5) as a non-commutativity: if we neglect the field ψ, the condition (2.5) is just the Heisenberg algebra and defines a non-commutative plane whose non-commutativity parameter is the FI parameter r. Thus the location of the vortex strings is ambiguous to the precision of $\mathcal{O}(r)$ in the z space, in terms of the vortex effective theory. It would be tempting to calculate the actual energy distribution of the vortex strings in the 3 dimensional space, given the string configuration defined by z in the vortex moduli space. This should be possible through the brane configuration and string theory, because intrinsically this map is the Seiberg-Witten map [34, 35], or in other words, Matrix theory multi-pole moments [36], which describes a coupling of non-commutative worldvolume configurations to bulk fields such as gravity. In [22, 23, 37] explicit map has been evaluated for various non-commutative configurations. To apply for our case of the vortex strings, we have to generalize the explicit Seiberg-Witten map [35] to the situation where the D-branes end on some other D-branes and hence additional fundamental fields are present.

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